Government of **Western Australia**
School Curriculum and Standards Authority

MATHEMATICS APPLICATIONS

Calculator-assumed

ATAR course examination 2018

Marking Key

Marking keys are an explicit statement about what the examining panel expect of candidates when they respond to particular examination items. They help ensure a consistent interpretation of the criteria that guide the awarding of marks.

2018/62742

Section Two: Calculator-assumed **65% (96 Marks)**

Question 8 **(7 marks)**

Anthony and Bryan each invest $4500 in accounts earning compound interest for a period of four years.

(a) Anthony places his money in an account earning interest at the rate of 3.24% per annum, compounded quarterly.

(i) Complete the table below, showing the value of Anthony's investment at the end of the second and third quarters. (2 marks)

Solution

Number of quarters money is invested	1	2	3	…	16
Value of investment ($)	4536.45	**4573.20**	**4610.24**	…	5120.00

Specific behaviours

✓ completes one correct value
✓ completes both correct values

(ii) State the recursive rule for Anthony's investment, which gives the values shown in the table above. (2 marks)

Solution

$T_{n+1} = T_n \times \left(1 + \frac{0.0324}{4}\right),\ T_0 = 4500$

or

$T_{n+1} = T_n \times \frac{0.0324}{4} + T_n,\ T_0 = 4500$

Specific behaviours

✓ states correct initial value
✓ states correct multiplier

(b) Bryan places his money in an account earning interest daily. After four years, the value of both Anthony's and Bryan's investments is the same.

Explain how the change to the compounding period has affected the annual rate of interest required for the value of Bryan's investment to be the same as that of Anthony. Include calculations to support your answer. (3 marks)

Solution
$5120 = 4500\left(1+\frac{x}{36500}\right)^{(365\times4)}$ $x = 3.227\%$ $x \approx 3.23\%$ p.a. Therefore, increasing the compounding period to daily reduces the required interest rate.
Specific behaviours
✓ shows correct calculation ✓ states the required interest rate ✓ states the correct effect on the interest rate

Question 9 **(8 marks)**

Deborah is purchasing mealworms for her pet lizard, Lizzy, to eat.

Deborah starts by buying 50 mealworms. She then buys an additional 15 at the start of each subsequent week. She feeds 12 mealworms to Lizzy each week, and each week a certain percentage of the mealworms dies.

Deborah has found that the approximate number of mealworms at the start of the n^{th} week can be modelled by M_n, where $M_{n+1} = 0.9(M_n - 12) + 15, \quad M_1 = 50$.

(a) What percentage of the mealworms dies each week? (1 mark)

Solution
10%
Specific behaviours
✓ states correct percentage

(b) Determine the approximate number of mealworms Deborah has at the start of the fifth week. (1 mark)

Solution
$a_{n+1}=0.9\cdot(a_n-12)+15$ n: 1, 2, 3, 4, 5, 6 a_n: 50, 49.2, 48.48, 47.832, 47.249, 46.724 48 mealworms
Specific behaviours
✓ states correct value

(c) Deborah claims that she will never run out of mealworms using this model. Justify her claim. (2 marks)

Solution
Solve $M = 0.9(M - 12) + 15$ $M = 42$ Deborah will always have 42 mealworms in the long run. **or** The sequence stabilises at 42.
Specific behaviours
✓ identifies a steady-state solution ✓ states that Deborah will always have 42 mealworms

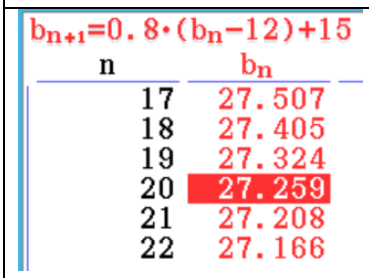
After 10 weeks, hot weather results in a larger percentage of the mealworms dying, so Deborah alters the model to:

$$N_{n+1} = 0.8\left(N_n - 12\right) + 15,\ N_1 = c$$

(d) (i) Determine the value of c. (1 mark)

Solution
$c = 45$
Specific behaviours
✓ determines the correct value of c

(ii) Determine the approximate number of mealworms Deborah has at the start of the thirtieth week. (1 mark)

Solution
 27 mealworms
Specific behaviours
✓ states correct value

Deborah's vet recommends feeding Lizzy 10 mealworms a week. She would also like to maintain a constant number of 30 mealworms at the start of each week, so she changes the above model to:

$$P_{n+1} = 0.8\left(P_n - 10\right) + k$$

(e) Determine the value of k, the number of mealworms she must buy each week, to ensure this occurs. (2 marks)

Solution
Solve $30 = 0.8\left(30 - 10\right) + k$ $k = 14$
Specific behaviours
✓ uses steady-state solution ✓ correctly calculates the value of k

Question 10 **(8 marks)**

(a) Determine the values of **A** and **B** for the Victorian data. (2 marks)

Solution
A = 27065 – (11985 + 9575) = 5505 **B** = 100 – (35.4 + 20.3) = 44.3%
Specific behaviours
✓ correctly calculates the value of **A** ✓ correctly calculates the value of **B**

(b) Compare the percentage of the total new vehicle sales in Western Australia with those in South Australia. (3 marks)

Solution
WA: $\frac{8026}{98763} \times 100 = 8.1265\%$ SA: $\frac{6464}{98763} \times 100 = 6.54\%$ Western Australia has a higher percentage than South Australia.
Specific behaviours
✓ identifies the correct value for the numerators ✓ calculates the correct percentages for both states ✓ comments on the association between states

(c) Describe the association between the number of sales of new passenger vehicles and new sports utility vehicles in Australia. (1 mark)

Solution
The number of new passenger vehicles sold is always higher than the number of sports utility vehicles sold.
Specific behaviours
✓ describes the correct association

(d) Compare and comment on the percentage sales of vehicles in the Northern Territory with those in other States/Territories. (2 marks)

Solution
The percentages of passenger vehicles **and** sports utility vehicles sold in the Northern Territory is the lowest in Australia. The percentage of other vehicles sold in the Northern Territory is the highest in the country.
Specific behaviours
✓ correctly compares passenger vehicle and sports utility vehicle sales to other states ✓ correctly compares other vehicle sales to other states

Question 11 **(8 marks)**

(a) Charles invests his money in an account earning interest at the rate of 3.35% per annum, with interest calculated and added to his account at the end of each month. He also deposits an additional amount of money at the end of each month. Determine the monthly deposit required by Charles if he is to reach his goal by his 65th birthday.

(3 marks)

Solution
N = 60 I% = 3.35 PV = – 465000 PMT = FV = 675000 P/Y = 12 C/Y = 12 Monthly deposit = $1921.80 Compound Interest N 60 I% 3.35 PV −465000 PMT −1921.801011 FV 675000 P/Y 12 C/Y 12
Specific behaviours
✓ correctly uses 60 months ✓ correctly uses P/Y and C/Y both 12 ✓ correctly determines the required monthly deposit

(b) (i) Determine the number of years that he will be able to receive this annuity.

(3 marks)

Solution
N = I% = 3.25 PV = – 675000 PMT = 65000 FV = 0 P/Y = 1 C/Y = 12 N = 12.92 Therefore, the fund will last 12 years. Compound Interest N 12.92274675 I% 3.25 PV −675000 PMT 65000 FV 0 P/Y 1 C/Y 12
Specific behaviours
✓ correctly uses payments of $65 000 and interest rate of 3.25% ✓ correctly uses P/Y equal to 1 ✓ correctly states N (rounded down to 12 years)

Question 11 (continued)

(ii) Charles is hopeful that it will be possible for him to continue receiving an annuity until his 85th birthday. He decides to find an alternative fund offering a different interest rate, while continuing to withdraw $65 000 each year. What annual interest rate would he need to receive to make his money last until his 85th birthday? (2 marks)

Solution
N = 20 I% = PV = – 675000 PMT = 65000 FV = 0 P/Y = 1 C/Y = 12 Interest rate = 7.03% Compound Interest N 20 I% 7.027477929 PV –675000 PMT 65000 FV 0 P/Y 1 C/Y 12
Specific behaviours
✓ correctly uses N = 20 ✓ correctly determines required interest rate

Question 12 **(6 marks)**

(a) What is the purpose of calculating moving averages for time series data? (1 mark)

Solution
To smooth out time series data. **or** To identify the trend.
Specific behaviours
✓ correctly states a valid reason for calculating moving averages

(b) Determine the values **A**, **B** and **C** in the above table. (3 marks)

Solution
$\frac{864+834+A}{3}=838,\ \therefore A=816$ $B=\frac{828+918+927+879+852}{5}=880.8$ $C=\frac{0.5\times840+927+936+894+867+828+0.5\times918}{6}=888.5$
Specific behaviours
✓ correctly determines A ✓ correctly determines B ✓ correctly determines C

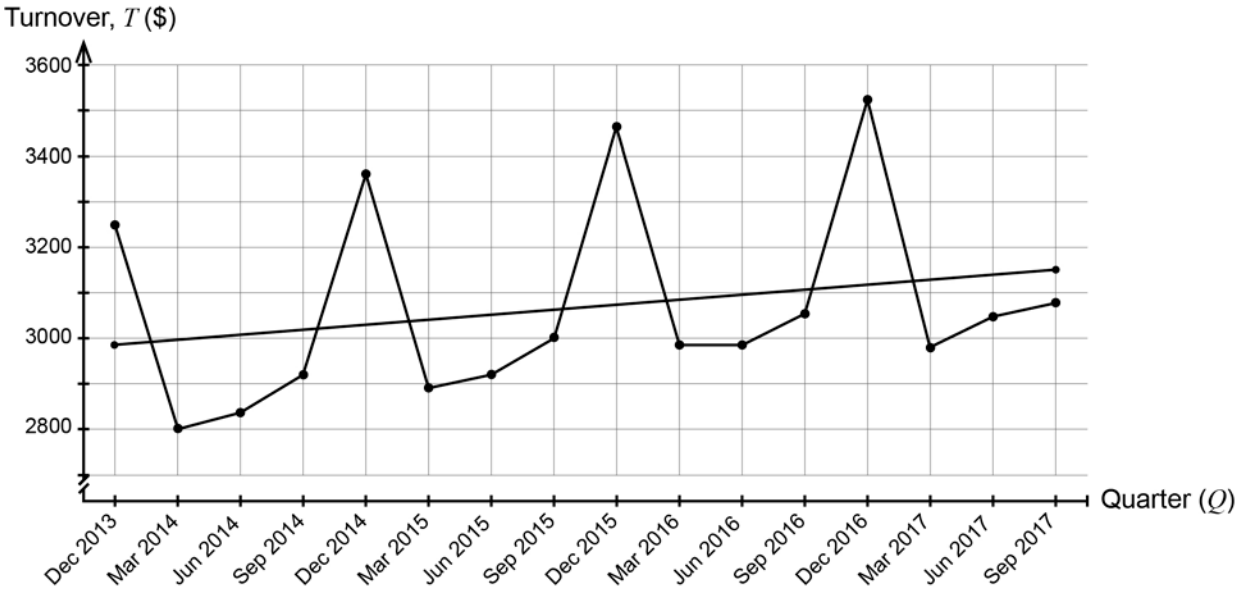
(c) From those in the table above, which is the most appropriate moving average for the manager of the service centre to consider? Justify your choice. (2 marks)

Solution
The most appropriate is the 5-point moving average. The data has a 5-point cycle. **or** The values in the 5-point moving average column are continually decreasing.
Specific behaviours
✓ correctly states the most appropriate moving average ✓ gives a valid justification

Question 13 **(13 marks)**

The graph below shows the quarterly retail turnover per capita ($) in Australia, i.e. the average amount spent per person at retail outlets during each quarter.

Quarterly retail turnover per capita, Australia, 2013–2017



The data for the next four quarters are shown in the following table.

Quarter	December 2016	March 2017	June 2017	September 2017
Quarterly retail turnover per capita ($)	3521.40	2980.10	3045.00	3075.30

(a) Complete the time series plot by including this additional information. (2 marks)

Solution
See graph above.
Specific behaviours
✓ correctly plots at least two points ✓ correctly plots all points and joins them

(b) The equation of the least-squares line for the above data is $T = 9.6143Q + 2986.50$, where Q = 1 for December 2013, Q = 2 for March 2014, etc.

(i) Fit this line to the graph. (2 marks)

Solution
See graph above.
Specific behaviours
✓ shows correct value for Q = 1 on the graph ✓ shows correct slope

(ii) Describe the trend and seasonality of this data. (2 marks)

Solution
There is an increasing/upwards trend. The high points are in the December quarter and the low points are in the March quarter.
Specific behaviours
✓ correctly states the overall trend ✓ correctly refers to high and low seasons

(c) The 4-point centred moving average for March 2017 is \$3152.78 (correct to two decimal places). Determine the **actual** retail turnover per capita for September 2016. (2 marks)

Solution
$3152.78 = \dfrac{0.5x + 3521.40 + 2980.10 + 3045.00 + 0.5 \times 3075.30}{4}$ $x = 3053.94$
Specific behaviours
✓ correctly uses March 2017 as the central figure ✓ determine the value for September 2016

(d) The seasonal indices (correct to two decimal places) are shown in the table below.

Quarter	Seasonal index
December	110.76%
March	95.00%
June	
September	98.20%

(i) Complete the table by determining the seasonal index for June. (1 mark)

Solution
June = $400 - (110.76 + 95.00 + 98.20)$ = 96.04%
Specific behaviours
✓ correctly determines the seasonal index for June

(ii) Use the seasonal index to determine the deseasonalised retail turnover per capita for December 2016. (2 marks)

Solution
$\dfrac{3521.40}{1.1076} = 3179.31$
Specific behaviours
✓ uses the correct seasonal index, as a decimal ✓ divides 3521.40 by the seasonal index to give the deseasonalised value

Question 13 (continued)

(iii) The deseasonalised retail turnover per capita for March 2016 is \$3142.42.
Determine the **actual** retail turnover per capita for this quarter. (2 marks)

Solution
$\frac{x}{0.95} = 3142.42$ $x = 2985.30$
Specific behaviours
✓ uses the correct seasonal index, as a decimal ✓ determines the correct value for March 2016

Question 14 **(12 marks)**

Marco is a plumber. Three years ago, he purchased a vehicle costing $48 000 for his business. He paid a deposit of $5000 and acquired a personal loan for the remainder from a financial institution, at a reducible interest rate of 22.5% per annum, compounded monthly. He agreed to make repayments of $1000 at the end of each month.

(a) (i) Use a recurrence relation to determine the amount Marco currently owes on the loan. (3 marks)

Solution
$T_{n+1} = \left(1 + \frac{22.5}{1200}\right) T_n - 1000,$
$T_{n+1} = 1.01875 T_n - 1000,\ T_0 = 43000$
$T_{36} = 33164.78$
He still owes $33 164.78.
Specific behaviours
✓ correctly determines a recurrence relation
✓ correctly uses $43 000 as the initial value
✓ correctly determines amount still owing after three years

(ii) Determine how much longer it will take him to completely pay off the loan. (2 marks)

Solution
$T_{89} = -649.89$
$89 - 36 = 53$
It will take an extra 53 months to pay off the loan.
Specific behaviours
✓ correctly determines total number of payments required
✓ correctly determines number of extra months

(b) After three years, Marco finds that his vehicle is only worth $27 150. Determine the average rate of depreciation of his vehicle, expressed as a percentage. (2 marks)

Solution
$27150 = 48000 \times x^3$
$x = 0.827$
$100 - 82.7 = 17.3\%$
Specific behaviours
✓ correctly solves equation to determine the ratio (0.827)
✓ correctly determines the rate of depreciation

Question 14 (continued)

(c) When Marco originally took out a personal loan for the purchase of his vehicle, he was given two options by the financial institution. These were:

- increasing his monthly repayment by $200, or
- taking an option of reducing the interest rate to 18.5% and maintaining repayments of $1000 per month.

In terms of time taken to pay off the loan and total paid for his vehicle, which should he have chosen and why? (5 marks)

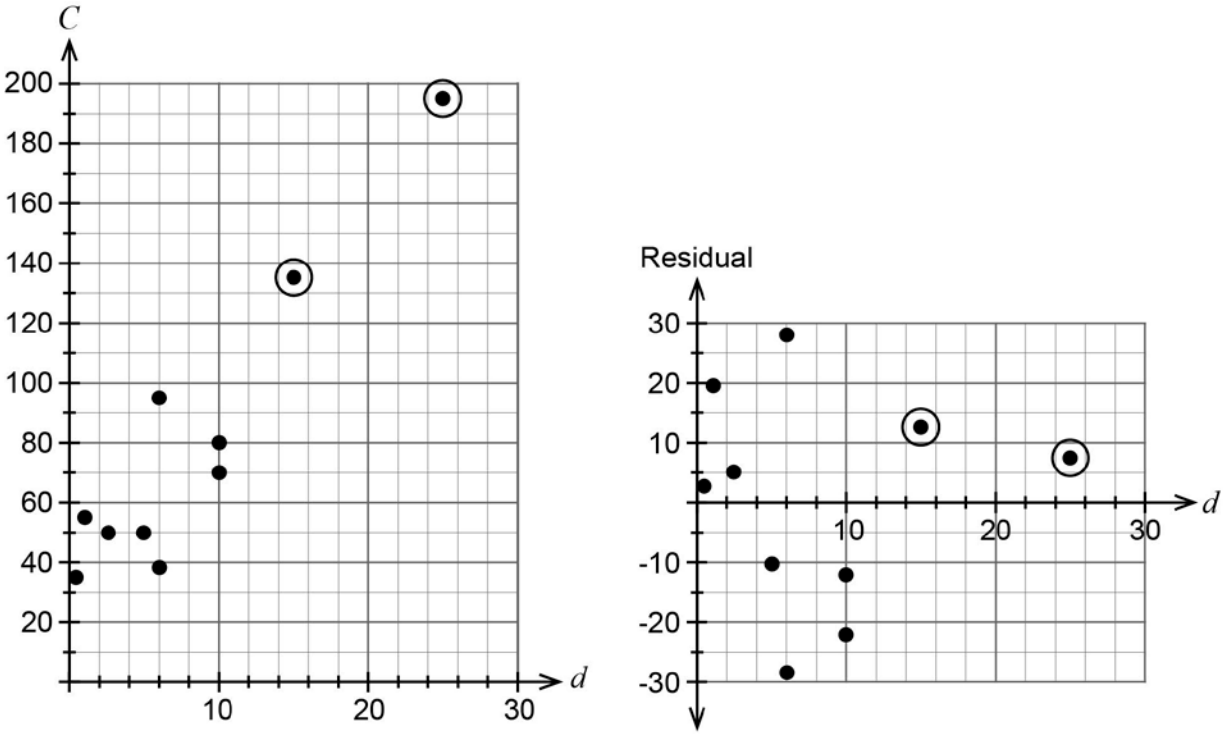
Solution	
Payment = $1200/mth Interest = 22.5% n = 59.99 ≈ 60 months Total to repay = 70 800 + (1200 – 14.24) = $71 985.76 (+ 5000)	Payment = $1000/mth Interest = 18.5% n = 71.08 ≈ 72 months Total to repay = 71 000 + (1000 – 921.44) = $71 078.56 (+ 5000)
He should have chosen the reduced interest rate as he would have paid less for the car. **or** He should have chosen to increase his repayments as he would have paid off the loan sooner and it would have cost only an extra $907.	
Specific behaviours	
✓ correctly determines time to repay first option ✓ correctly determines time to repay second option ✓ correctly determines total repayment for first option ✓ correctly determines total repayment for second option ✓ gives a valid reason for choosing an option	

Question 15 **(12 marks)**

Ali is researching mobile phone carriers and has found several plans with monthly contracts. The table below shows the data allowance, GB (d) and the monthly cost \$ (C), of ten plans that he is considering.

Data allowance GB (d)	10	2.5	0.5	15	5	1	6	6	25	10
Monthly cost \$ (C)	70	50	35	135	50	55	95	38	195	80

The graphs below show a scatterplot and a residual plot for the information in the table, with two points missing on both graphs.



(a) Plot the two missing points on the scatterplot. (2 marks)

Solution
See graph above.
Specific behaviours
✓ plots (15,135) correctly
✓ plots (25,195) correctly

(b) (i) Determine the equation of the least-squares line for the information in the table and state the correlation coefficient. (2 marks)

Solution
$C = 6.30d + 29.25$ and $r = 0.932$
Specific behaviours
✓ correctly determines the least-squares line ✓ correctly determines the correlation coefficient

Question 15 (continued)

(ii) Describe the linear association between Data allowance and Monthly cost. (2 marks)

Solution
Positive and strong as the correlation coefficient is strong
Specific behaviours
✓ states the association is positive ✓ states the association is strong

(iii) Approximately how much does the cost change for every additional GB of data allowance? (1 mark)

Solution
Changes by $6.30
Specific behaviours
✓ states the correct change

(iv) What percentage of the variation in monthly cost can be explained by the variation in the data allowance? (1 mark)

Solution
Approximately 87%
Specific behaviours
✓ states correct percentage

(c) (i) Calculate the two missing residuals and include them on the residual plot. (2 marks)

Solution
(15, 11.21), (25, 8.19)
Specific behaviours
✓ calculates correct residuals ✓ correctly plots residuals on graph

(ii) What feature of the residual plot indicates that a linear model would be appropriate for the data? (1 mark)

Solution
Plots are random, i.e. no pattern evident.
Specific behaviours
✓ states correct reason

(d) Predict the monthly cost of a plan with a data allowance of 20 GB. (1 mark)

Solution
$155.30
Specific behaviours
✓ states correct cost

Question 16 **(11 marks)**

Natalia inherits a sum of money from her grandfather. She wishes to place it in a high-interest savings account.

She is considering the following two options:

Account A: interest rate 4.40% per annum, compounded monthly
Account B: interest rate 4.30% per annum, compounded daily.

(a) The effective annual interest rate for Account A is 4.49% (correct to two decimal places). Determine the effective annual interest rate for Account B. (1 mark)

Solution
$i = \left(1 + \frac{0.043}{365}\right)^{365} - 1 = 0.0439$ i.e. 4.39%
Specific behaviours
✓ correctly calculates the effective interest rate for Account B

Natalia's bank offers her another account, C, with an interest rate of 4.50% per annum.

(b) Under what circumstances will this interest rate and the effective annual interest rate be the same? (1 mark)

Solution
If the interest is compounded annually.
Specific behaviours
✓ states correct reason

(c) Which account (A, B or C) should Natalia choose to maximise her savings? Explain your reasoning. (2 marks)

Solution
Account C as it has the highest effective interest rate.
Specific behaviours
✓ correctly states Account C ✓ gives valid reason

Question 16 (continued)

Natalia's sister, Elena, has inherited $25 000 from her grandfather. She decides to invest this money in a high-interest savings account, with interest compounded monthly. Elena also chooses to deposit an additional $250 into this account at the end of each month.

The table below shows Elena's account balance over the first three months.

Month	Account balance at start of month	Interest earned	Deposit	Account balance at end of month
1	$25 000.00	$125.00	$250.00	$25 375.00
2	$25 375.00	$126.88	$250.00	$25 751.88
3	$25 751.88	$128.76	$250.00	$26 130.64

(d) Show that the annual interest rate that applies to Elena's account is 6%. (1 mark)

Solution
$r = \frac{125}{25000} \times 12 \times 100 = 6\%$
Specific behaviours
✓ correctly shows the annual interest rate is 6%

(e) The amount in Elena's account, A_n at the end of month n, can be expressed as a recursive rule, $A_{n+1} = cA_n + d$, $A_0 = 25\ 000$. Determine the values of c and d. (2 marks)

Solution
$c = 1.005$, $d = 250$
Specific behaviours
✓ correctly states the value of c ✓ correctly states the value of d

(f) After two years, Elena wishes to use the money she has saved as a deposit for a house. An amount of $35 000 will be required. Unfortunately, Elena has realised that by depositing $250 each month she will **not** reach her savings goal.

(i) If she only deposits $250 each month, by how much will she be short of the required deposit? (2 marks)

Solution
$A_{24} = 34536.98$ $35000 - 34536.98$ $= 463.02$ After two years Elena has $34 536.98, so she will need an extra $463.02.
Specific behaviours
✓ correctly determines the amount saved after 2 years ✓ correctly calculates the extra amount needed

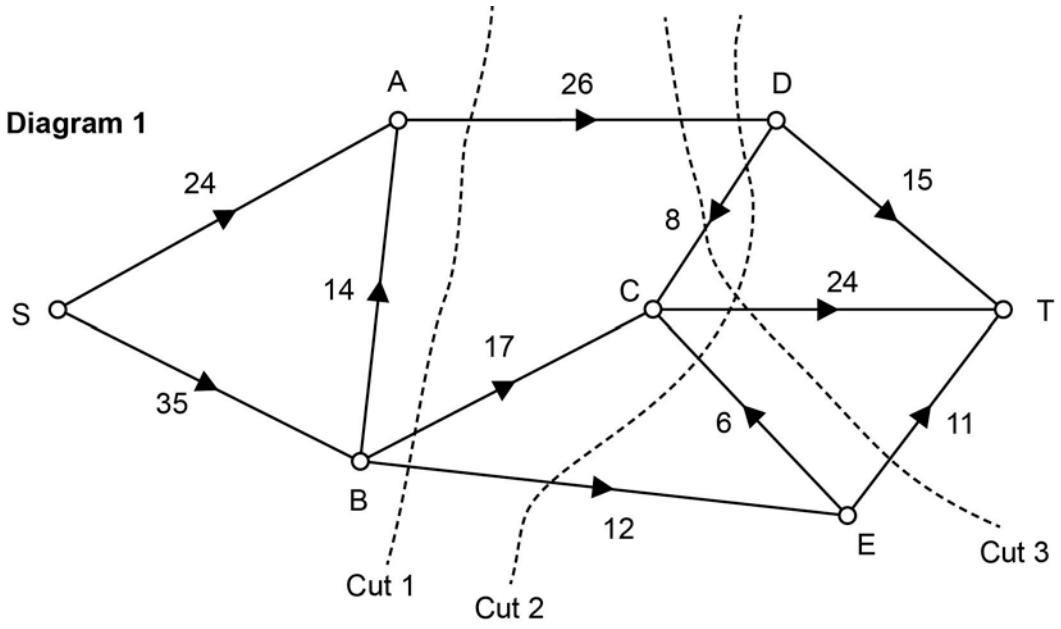
(ii) What increase in the monthly deposit is required for Elena to save the $35 000 in two years? (2 marks)

Solution	
Elena would have needed an extra $18.21 per month	N 24 I% 6 PV 25000 PMT 268.2061025 FV −35000 P/Y 12 C/Y 12
Specific behaviours	
✓ correctly determines the new deposit amount ✓ correctly calculates the extra amount needed	

Question 17 **(11 marks)**

Diagram 1 shows a network of pipes. The number on each edge gives the capacity of that pipe in L/min.

Diagram 1

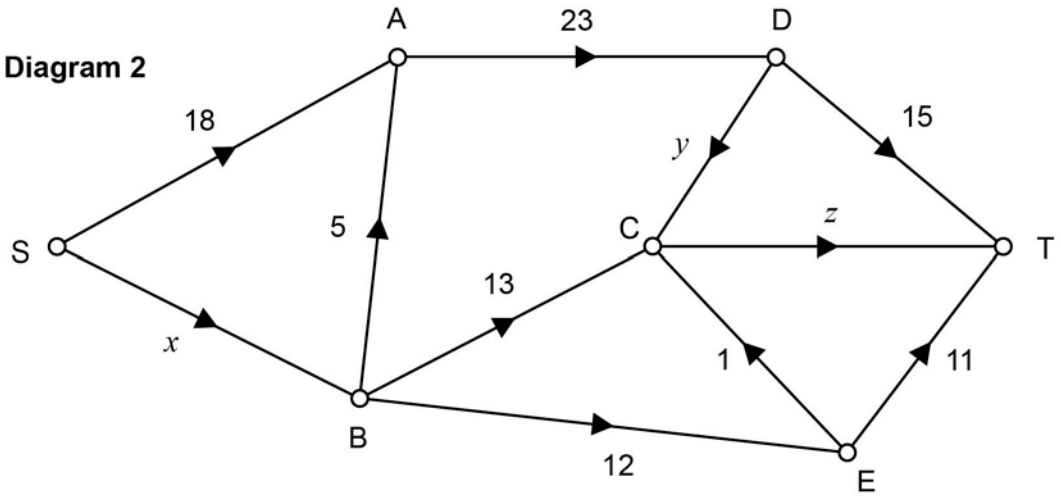


(a) State the capacities of the three cuts in Diagram 1. (3 marks)

Solution
Cut 1 : 12 + 17 + 26 = 55 L/min Cut 2 : 12 + 24 + 26 = 62 L/min Cut 3 : 11 + 24 +26 = 61 L/min
Specific behaviours
✓ correctly states the capacity of Cut 1 ✓ correctly states the capacity of Cut 2 ✓ correctly states the capacity of Cut 3

Diagram 2 shows a possible flow for the network of pipes.

Diagram 2



(b) (i) Explain why the value of x is 30. (1 mark)

Solution
The flow into a node must equal the flow out of that node.
Specific behaviours
✓ states correct reason

(ii) Calculate the values of y and z. (2 marks)

Solution
$18+x=15+11+z \therefore z=22$ $13+1+y=z \therefore y=8$
Specific behaviours
✓ correctly calculates the value of z ✓ correctly calculates the value of y

(c) State which of the pipes are at full capacity in Diagram 2. (2 marks)

Solution
DT, ET, BE and DC
Specific behaviours
✓ correctly gives at least 2 correct responses ✓ correctly gives all correct responses

(d) State the value of the flow for the network in Diagram 2. (1 mark)

Solution
18 + 30 = 48 or 15 + 22 + 11 = 48 L/min
Specific behaviours
✓ correctly states the flow

Question 17 (continued)

(e) (i) The value of the flow for Diagram 2 can be increased by 2 L/min. List the series of pipes that could be used to achieve this. (1 mark)

Solution
SBCT by an increase of 2 = 50
Specific behaviours
✓ correctly states the route

(ii) Show that the increased flow in part (e)(i) is a maximum for this network of pipes. (1 mark)

Solution
Minimum cut through DT, CT and ET = 50 on Diagram 1 OR DT, CT and ET are now at full capacity after the increase in flow from (e)(i)
Specific behaviours
✓ correctly shows the increased flow is a maximum for this network of pipes

ACKNOWLEDGEMENTS

Question 13 Graph data source adapted from: Australian Bureau of Statistics. (2018). *Table 19: Quarterly retail turnover per capita, Australia, all series*. Retrieved April, 2018, from http://www.abs.gov.au/AUSSTATS/abs@.nsf/DetailsPage/8501.0Nov%202017?
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